MODELING AND APPLICATION OF GRID-REFINED FLOOD INUNDATION IN TAIWAN

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ABSTRACT

A local grid refinement method is adopted herein to efficiently obtain detailed inundation information for local area. First, coarse grids are adopted for global area simulation to determine the boundary conditions of local area during a coarse time step. The local area is then solved using fine grids within the same coarse time step. As a result, the variables of refined coarse grids are updated with the results of fine grids before advancing to next time step. The coarse and fine grids are applied alternately to speed up the simulation processes.

A study case is employed here for comparing the accuracy of the local grid-refined method with the uniform fine grid method. The statistical analysis shows that the root mean square error (RMSE) is 0.023m and the R squared (R^2) is 0.995, which indicates that the local grid-refined method has good accuracy and takes much less computation time. An example application of analyzing flood and inundation is conducted for the Miaoli station, Taiwan High Speed Rail. The simulated results and suggested measures can be applied to map out the inundation potentials, and to support the design of Taiwan High Speed Rail.

Keywords: Flood inundation model, Local grid-refined, Inundation, Coarse grid, Fine Grid

INTRODUCTION

In the simulation of two-dimensional flood inundation flow with uniform grids, it is inefficient to simulate the entire domain with fine resolution of grids. The depth-averaged shallow water equations are widely adopted for traditional two-dimensional flood inundation models, which require specified boundary conditions that the fluxes only enter or leave the simulation domain on several control points. Thus, the watershed boundaries or levees that have no fluxes through over are commonly used. In most cases, only the inundation potential for a specified designed rainfall on the local regions of the watershed area interests us, for example, a town or industrial area surrounded by farms or plains. The local regions usually have no independent boundary conditions for the specified flow fluxes. Therefore, the simulation domain must be enlarged enough to have proper boundary conditions.

The enlarged global simulation domain is often several times larger than the local region of interest. Accordingly, more computing resources like hardware requirements and computation time are needed if the entire domain is precisely simulated with uniform fine grids. Larger the simulation area is, more the number of grid points and longer computation time are needed. But the only information we concern about is the inundation in the local region, where usually has high population density or well-developed economical activities, but small proportion in area. And so, to spend most resources for obtaining the information we are not really interested in is inefficient.

Consequently, a local grid-refined method is developed, which computes the global domain and local region with different resolutions of space and time. In other words, coarser grid and larger time step are used in global domain, but finer grid and smaller time step are chosen in local region computing for precision consideration. The boundary conditions of local region are provided by the global computation. Meanwhile, the global domain is also patched with the local region computing results.

Adaptive methods have been used in many classical problems of numerical analysis. Dwyer et al. (1980,1984), Berger and Oliger (1984) have done adaptive finite difference calculations in one or two space dimensions. Ewing et al. (1990) also conducted many studies and developed finite difference scheme for parabolic problems on composite grids with refinement in both time and space. In this paper, we applied the method for solving the flood inundation problems.

GRID-REFINED FLOOD INUNDATION MODEL

Two-Dimensional Floods Inundation Model

Based on the assumption that the acceleration of water flow on land surface is small compared with the gravitation and friction, the depth-averaged shallow water equations on land surface can be written as:

$$\frac{\partial d}{\partial t}+\frac{\partial[(1-\beta)ud]}{\partial x}+\frac{\partial[(1-\beta)vd]}{\partial y}=q \qquad (1)$$

$$-\frac{\partial h}{\partial x}=S_{fx}+\frac{qu}{dg} \qquad (2)$$

$$-\frac{\partial h}{\partial x}=S_{fy}+\frac{qv}{dg} \qquad (3)$$

Where d = depth of flow, h = water stage, u = velocity component in the x-direction, v = velocity component in the y-direction, t = time, g = gravitational acceleration, q = source or sink per unit area, n = Manning's roughness coefficient. $S_{fx}=\frac{n^2u\sqrt{u^2+v^2}}{d^{4/3}}$ and $S_{fy}=\frac{n^2v\sqrt{u^2+v^2}}{d^{4/3}}$ are the friction slope in the x-direction and the y-direction, respectively.. $\beta=\sqrt{\frac{A_b}{A}}$ is the detaining ratio that represents a linear ratio of building area to the total area of interest. For computational convenience, we further assume that $|u|$ and $|v|$ is used instead of $\sqrt{u^2+v^2}$ for computing the friction slope S_{fx} and S_{fy}, respectively.

For the present study, a two-step alternating direction explicit (ADE) method is used. The ADE is a finite difference numerical scheme, which solves the shallow water equations with a uniform grid structure. The numerical scheme uses a non-staggered grid structure. The variables of depths are located on the centers of grids and the velocities are on the interfaces of grids.

The finite different equations derived from Eqs. (1) to (3) in each time step are:

For the first time step$(m+\frac{1}{2})$:

$$d_{i,j}^{m+\frac{1}{2}}=d_{i,j}^{m}-\frac{\Delta t}{2}\left\{\frac{(1-\beta)\left[\left(d^m u^{m+\frac{1}{2}}\right)_{i+\frac{1}{2},j}-\left(d^m u^{m+\frac{1}{2}}\right)_{i-\frac{1}{2},j}\right]}{\Delta x}+\frac{(1-\beta)\left[(dv)_{i,j+\frac{1}{2}}^{m}-(dv)_{i,j-\frac{1}{2}}^{m}\right]}{\Delta y}-q_{i,j}^{m+\frac{1}{2}}\right\} \qquad (4)$$

$$\frac{\left[(d+z)_{i,j}^{m+\frac{1}{2}}-(d+z)_{i+1,j}^{m+\frac{1}{2}}\right]}{\Delta x}=\left\{u^{m+\frac{1}{2}}\left[\frac{(n)^2\,|u^{m+\frac{1}{2}}|}{(d^m)^{\frac{4}{3}}}+\frac{q^{m+\frac{1}{2}}}{(d^m g)}\right]\right\}_{i+\frac{1}{2},j} \qquad (5)$$

$$\frac{\left[(d+z)_{i,j}^{m+\frac{1}{2}}-(d+z)_{i,j+1}^{m+\frac{1}{2}}\right]}{\Delta y}=\left\{v^{m+\frac{1}{2}}\left[\frac{(n)^2\,|v^{m+\frac{1}{2}}|}{\left(d^{m+\frac{1}{2}}\right)^{\frac{4}{3}}}+\frac{q^{m+\frac{1}{2}}}{\left(d^{m+\frac{1}{2}}g\right)}\right]\right\}_{i,j+\frac{1}{2}} \qquad (6)$$

Where m, i, and j are time, x, and y spatial indices, respectively, $\Delta t=t^{m+1}-t^m$, $\Delta x=x_{i+1}-x_i$, $\Delta y=y_{i+1}-y_i$. Using Eqs. (4) and (5), $u^{m+\frac{1}{2}}$ and $d^{m+\frac{1}{2}}$ along the x-direction can be solved simultaneously. Then $v^{m+1/2}$ can be found from Eq. (6).

For the second time step$(m+1)$:

$$d_{i,j}^{m+1}=d_{i,j}^{m+\frac{1}{2}}-\frac{\Delta t}{2}\left\{\frac{(1-\beta)\left[(du)_{i+\frac{1}{2},j}^{m+\frac{1}{2}}-(du)_{i-\frac{1}{2},j}^{m+\frac{1}{2}}\right]}{\Delta x}+\frac{(1-\beta)\left[\left(d^{m+\frac{1}{2}}v^{m+1}\right)_{i,j+\frac{1}{2}}-\left(d^{m+\frac{1}{2}}v^{m+1}\right)_{i,j-\frac{1}{2}}\right]}{\Delta y}-q_{i,j}^{m+1}\right\} \qquad (7)$$

$$\frac{\left[(d+z)_{i,j}^{m+1}-(d+z)_{i,j+1}^{m+1}\right]}{\Delta y}=\left\{v^{m+1}\left[\frac{(n)^2\,|v^{m+1}|}{\left(d^{m+\frac{1}{2}}\right)^{\frac{4}{3}}}+\frac{q^{m+1}}{\left(d^{m+\frac{1}{2}}g\right)}\right]\right\}_{i,j+\frac{1}{2}} \qquad (8)$$

$$\frac{\left[(d+z)_{i,j}^{m+1}-(d+z)_{i,j+1}^{m+1}\right]}{\Delta x}=\left\{u^{m+1}\left[\frac{(n)^2\,|u^{m+1}|}{(d^{m+1})^{\frac{4}{3}}}+\frac{q^{m+1}}{(d^{m+1}g)}\right]\right\}_{i+\frac{1}{2},j} \qquad (9)$$

In the second time step, Eqs. (7) and (8) are used to solve v^{m+1} and d^{m+1} along the y-direction and Eq. (9) is employed to solve u^{m+1} directly. The flood inundation model adopted in the study has been well calibrated and verified (Yen and Hsu, 1984; Hsu et al. 1990; Hsu et al, 1998).

Local Grid-refined Method

As mentioned earlier, the scheme of solving the global domain with uniform fine grid is inefficient to obtain the inundation information of local area. A local grid-refined method is hence employed. That is, the coarser grid is used to simulate the inundation of the global domain to get the boundary fluxes of local region, but the finer grid is chosen in local region computing for precision consideration.

Following the grid structure defined by Berger and Oliger (1984), the level of grids is defined as the number of coarser grids, where the fine grids are contained in. The grid level L=0 is the coarsest grid in the hierarchy. The sub-grid level L=1 indicates the refined grids within level 0, and the sub-grid L=2 is the refined grids within level 1, and so on. With such a nested structure, the method can be recursively applied for efficient computation.

For the grid level L and L+1, we first compute the coarser grid L as an upper level with a larger time step and thus obtain the boundary fluxes of the local grid-refined region for the finer grid L+1 as a lower level computation. Then, interpolate spatially to fit the boundary conditions of the lower level and progress the computation with a smaller time step to satisfy the numerical stability criteria. If there exists a finer grid level L+2 than current lower level, the level L+1 is set as the new upper level and L+2 as the new lower level. Repeat the interpolation and lower level computation procedures until no more finer grid level exists. When the finer grid computation forwards several time steps and the total of small time steps equals to the large time step of the upper level, the values of the variables for the upper level grid points which have been refined, are updated by the results of the lower level. The algorithm of the local grid-refined method is shown in Fig. 1.

The boundary fluxes of local grid-refined region are treated individually for each two contiguous coarser grids of level L on both sides of the coarse-fine interface. An interface with grid refinement is shown in Fig. 2-a. In Fig. 2-b, the grid on the right side hand of the interface is refined as level L+1 with a ratio r both in x and y directions. Therefore the grid sizes of the finer grid are $\Delta x_{L+1} = \frac{\Delta x_L}{r}$ and $\Delta y_{L+1} = \frac{\Delta y_L}{r}$. In addition, the smaller time step $\Delta t_{L+1} = \frac{\Delta t_L}{r}$ is also required for numerical stability consideration.

For simplified expression in following description, the flux, velocity and depth across or on the interface between the two coarse girds of level L are presented as F, $\overline{U}$, and $\overline{D}$. Meanwhile, the fluxes across the boundary of the refined grids of level L+1 are presented as f_k, k=1, 2, …, r.

A non-staggered grid structure is used in the numerical scheme, where the depth variables are located on the center of grid cells and the velocity variables are on the boundaries of grid cells. When the coarser grid was computed, the velocity $\overline{U}$ on the interface is directly available and the depth $\overline{D}$ is obtained from the average of the grid depths on both sides of the interface, $\overline{D} = \frac{1}{2}(D_{OUTER} + D_{INNER})$, where D_{OUTER} is the depth of the grid on the outside of the interface for refinement, and D_{INNER} is the depth of the grid on the inside of the interface. The flux per unit width, F, between two contiguous coarse grids across the interface is computed as

$$F = \overline{U} \cdot \overline{D} \quad \textbf{(10)}$$

$$\sum_{k=1}^{r} f_k \cdot \Delta y_{L+1} = F \cdot \Delta y_L \quad \textbf{(11)}$$

Thus, Eq. (11) can be further expressed as,

$$\sum_{k=1}^{r} f_k = F \cdot r \quad \textbf{(12)}$$

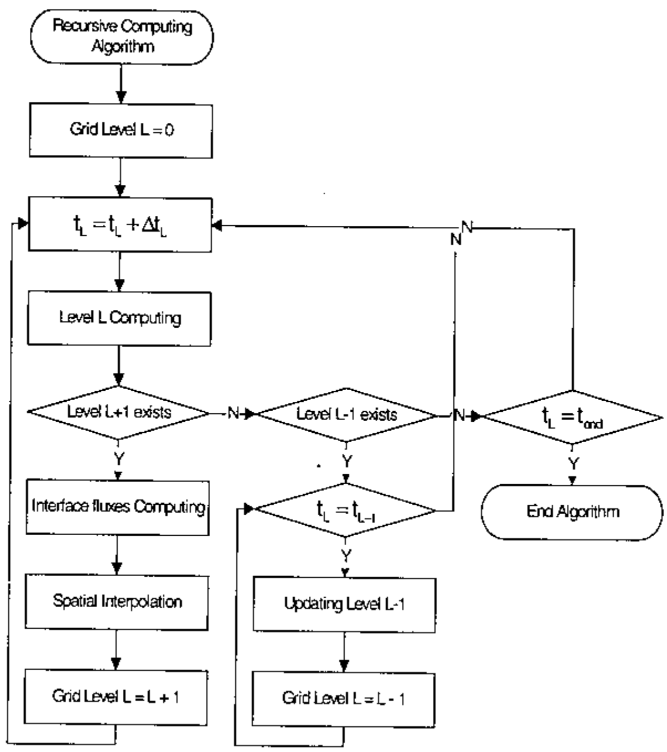
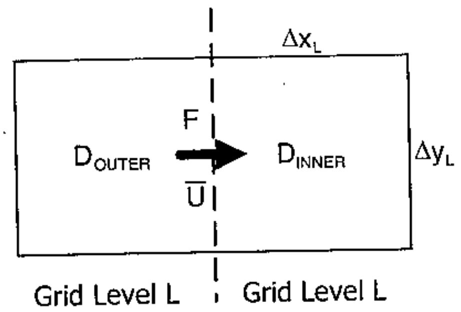
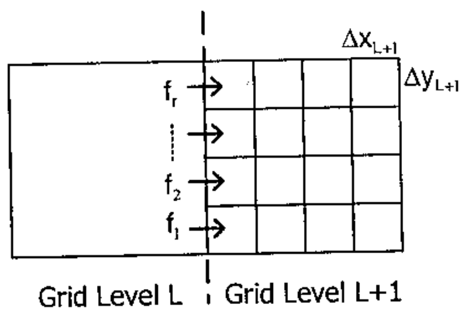


Fig. 1. Flowchart of the recursive local grid-refined algorithm



a. Coarse-coarse grid interface



b. Coarse-fine grid interface

Fig. 2. The interface with grid refinement

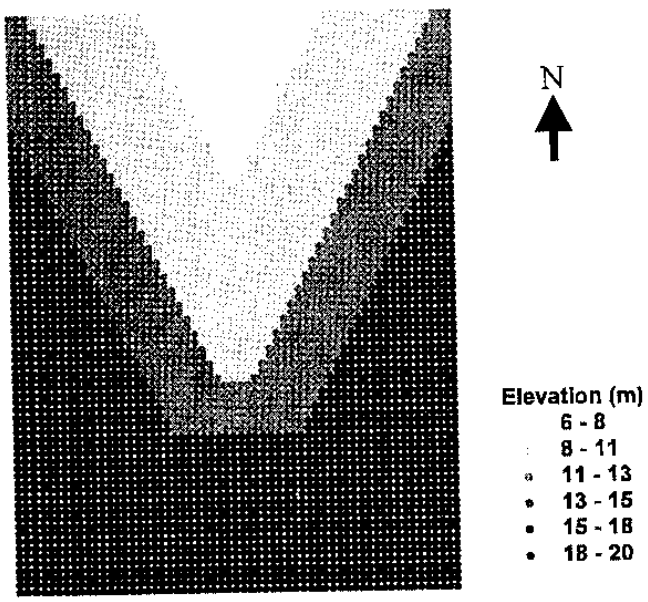


Fig. 3. Digital terrain model of study area

Besides, the flux redistributing establishes the relationships between the fluxes across the interface of the fine grid f_k, k=1, 2, …, r. The fluxes f_k, k= 2, 3, …, r are expressed as functions of f_1. In this paper, the flux of coarse grids is considered as equally distributed to the fine grids, which means that the boundary flux per unit with each fine grid is equal. Therefore, another r-1 equations $f_2 = f_1$, $f_3 = f_1$, …, $f_r = f_1$ are derived. Including Eq. (12), the r unknowns f_k, k=1, 2, …, r can be solved as functions of F and then used as boundary conditions of fine grid computation.

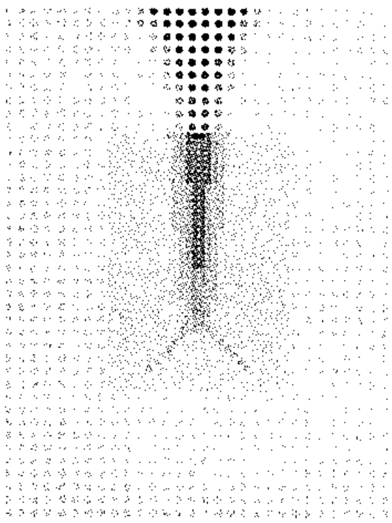
VERIFICATION

The model was applied to an ideal rectangular watershed of 1.2km x 1.6km shown in Fig. 3. The elevations in both eastern and western sides are higher and slope down to the middle with 0.001m/m gradient, and also dip toward north with 0.0005m/m gradient. The triangular southern area has a larger gradient 0.0015m/m in y-direction down to the north, but flat along the x-direction. Thus, it forms a Y-shaped village topography, and an outlet weir is set on the northern middle boundary. A 24-hour rainfall event with an intensity 50 mm/hr during the first 12 hours is simulated.

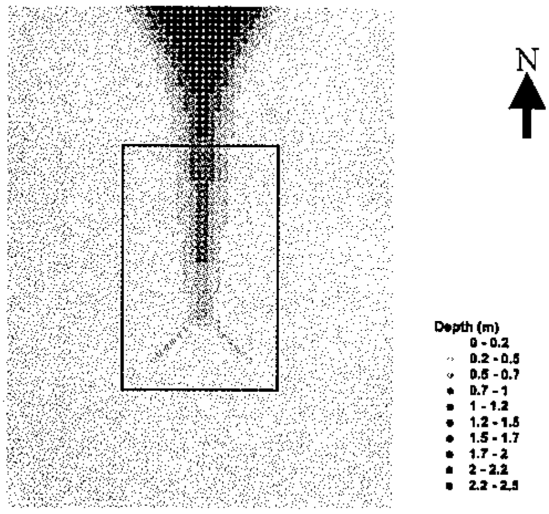
Two grid levels were adopted here, the coarse grid size 400m x 400m was used for global domain computation and the central region was refined with a ratio 2 for detailed simulation. The coarse grid had 1,200 points and 1 sec was chosen as the time step size for the coarse grid computation. The fine grid had 1120 points and 0.5 sec was used for fine grid computation. It took about 58mins for computing and the maximum inundation depths of each grid point are shown as Fig. 4.a. The depths in the central refined region varied from 0.019m to 1.010m.

A uniform fine grid method with a grid size of 200m x 200m, 4,800 grid points, and 0.5 sec as time step for global domain computation was also simulated for comparison and the result is shown in Fig. 4.b. It took 2hr 18mins for computation and the depths in the central refined region varied from 0.039m to 1.052m.

In Fig. 5, the inundation depths obtained from local grid-refined and uniform fine grid methods of each grid in the central refine region are compared. The statistical analysis shows that the bias (B) of the mean depth is 0.005m, the root mean square error (RMSE) is 0.023m and the R squared (R^2) is 0.995, the values indicate that the local grid-refined method has good accuracy and takes much less computation time.



a. Local grid-refined method



b. Uniform fine grid method

Fig. 4. The simulated inundation depth of study area

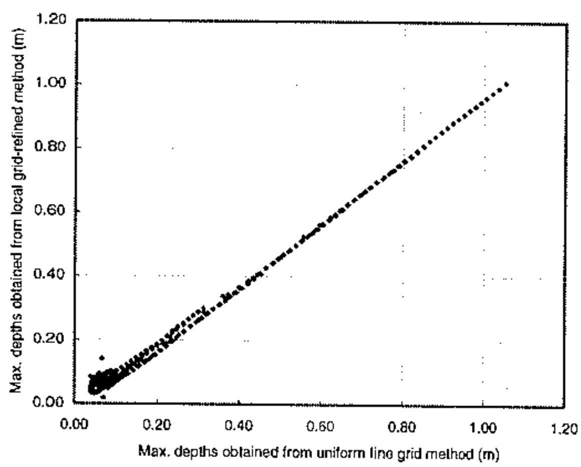


Fig. 5. The correlation for the results of uniform fine grid and local-grid

APPLICATION

An example application of analysis of flood and inundation is conducted in the Miaoli station of Taiwan High Speed Rail, located in the Chungkung and Holung Creek basins (shown in Fig. 6). The planned Taiwan High Speed Rail (THSR) running from Metropolitan Taipei city, the largest city and capital of Taiwan, with a population of 5.7 million, to Kaohsiung, the second largest city, is approximately 345 km in length and consists of seven stations and four depots. Such an important transportation system requires a high level of flood protection standard. According to the design criteria issued by the Ministry of Transportation in Taiwan, a rainfall of 200-year return period is the design basis for a station of THSR. The inundation map with 200-year return-period rainfalls and 24-hour duration at Miaoli station has been conducted.

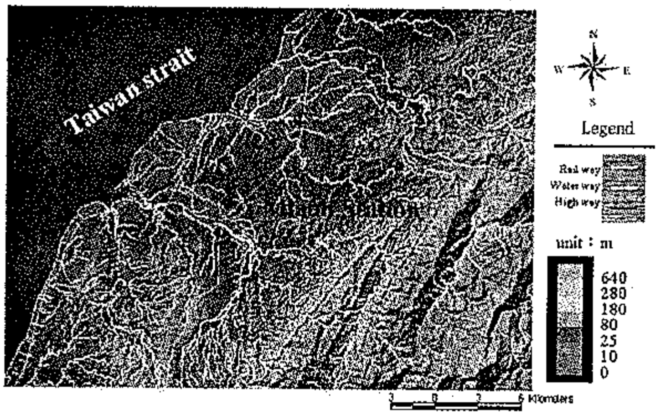
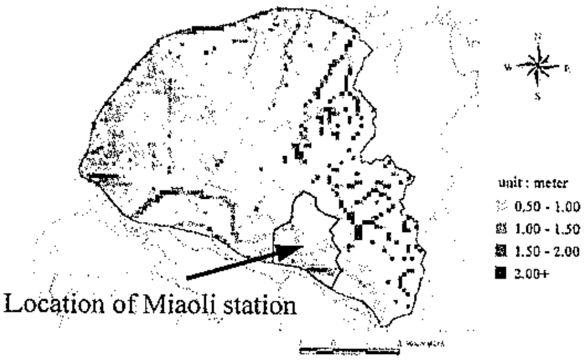


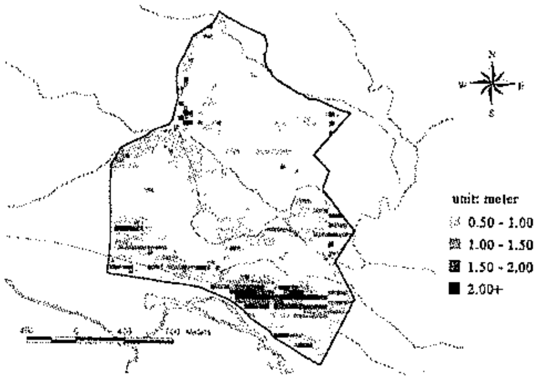
Fig. 6. Simulated area of Miaoli Station

Two grid levels were adopted here, the coarse grid size 120m x 120m was used for global domain computation for the middle part of Miaoli County and the Miaoli Station was refined with a ratio of 3 for detailed simulation. The coarse grid had 4,120 points and 1 sec was chosen as the time step size for the coarse grid computation. The fine grid had 5,800 points and 0.4 sec was used for fine grid computation. It took about 3.6 hr. for computing and the maximum inundation depths of each grid point are shown as Fig. 7. The depths in the Miaoli station with refined region grids from 2.5 m to 3.0 m.

In the southern part of Miaoli station, three inchoate suggested inundation - proofing measures, including raising facility elevation, building waterproofing exterior wall, and the combination of both, are proposed and investigated. The simulated results and suggested inundation-proofing measures can be applied to map out the inundation potential, and used as the design alternatives of Taiwan High Speed Rail.



a. Simulated results for global domain



b. Simulated results for Miaoli Station

Fig. 7. The simulated inundation depth of Miaoli Station

CONCLUSION

For the local region as a portion of large watershed area, the local grid-refined flood inundation model developed herein is capable of simulating the inundation efficiently and accurately.

By using different resolutions of grids, the computer efforts for the surrounding area where high precision is unnecessary can be saved. In addition, the result accuracy of local region is remained as well as that obtained using uniform fine grid method.

The simulated results of the application case can be applied to map out the inundation potentials, and used to support the design of Taiwan High Speed Rail.

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BIBLIOGRAPHY

Berger, M. J. and Oiger, J., 1984, Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations, J. Comp. Phys., V. 53, P. 484-512.

Dwyer, H. A., Kee, R. J. and Sanders B. R., 1980, Adaptive Grid Method for Problems in Fluid Mechanics and Heat Transfer, AIAA Journal, V.18, 10, P. 1205.

Dwyer, H. A., 1984, Grid Adaptation for Problems in Fluid Dynamics, AIAA Journal, V. 22, 12, P. 1705-1712.

Ewing, R. E., Lazarov, R. D., and Vassilevski, P. S., 1990, Finite Difference Schemes on Grids with Local Refinement in Time and Space for Parabolic Problems I. Derivation, Stability and Error Analysis, Computing, V. 45, P.193-215.

Hsu, M. H. et al., 1990, Two Dimensional Inundation Model for Taipei City, Fifth International Conference on Urban Storm Drainage, Osaka, Japan.

Hsu, M. H., Teng, W. S. and Wu, F. C., 1998, Inundation models for the Pa-chang Creek basin in Taiwan, Proceedings of the National Science Council, Republic of China, Part A: Physical Science and Engineering V. 22, 2 , P. 279-289.

Yen, C. L. and Hsu, M. H., 1984, Flood Routing Models for Tanshui River System Under Present Condition, Proceeding of the CCNAA-AIT Joint Seminar on Research for Multiple Hazards Mitigation, Natl. Cheng-Kung Univ. Tainan, Taiwan.